fourth GRADE

2018-2019 Guide

April 29-May 31

<u>Eureka</u>

Module 4: *Angle Measurement and Plane Figures*

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Module 4 Performance Overview

- Topic A begins with students drawing points, lines, line segments, and rays and identifying these in various contexts and familiar figures. As they continue, students recognize that two rays sharing a common endpoint form an angle. They also draw acute, right, and obtuse angles. This represents students' first experience with angle comparison and the idea that one angle's measure can be greater (obtuse) or less (acute) than that of a right angle. Next, students use their understanding of angles to explore relationships between pairs of lines, defining and recognizing intersecting, perpendicular, and parallel lines.

- In Topic B, students explore the definition of degree measure. Students divide the circumference of a circle into 360 equal parts, with each part representing 1 degree. Students apply this understanding as they discover that a right angle measures 90° and, in turn, that the angles they know as acute measure less than 90°, and obtuse angles measure more than 90°. Students discover that an angle can be seen as a measure of turning. This reasoning forms the basis for the understanding that degree measure is not a measure of length.

- In Topic C, students use concrete examples to discover the additive nature of angle measurement. As they work with angles, students see that the measures of all of the angles at a point, with no overlaps or gaps, add up to 360°. Students use what they know about the additive nature of angle measure to reason about the relationships between pairs of adjacent angles. Students discover that the measures of two angles on a straight line add up to 180° (supplementary angles) and that the measures of two angles meeting to form a right angle add up to 90° (complementary angles).

- An introduction to symmetry opens Topic D. Students recognize lines of symmetry for two dimensional figures, identify line-symmetric figures, and draw lines of symmetry. They then classify triangles as right, acute, or obtuse based on angle measurements. They also learn that triangles can be classified as equilateral, isosceles, or scalene based on side lengths. For isosceles triangles, lines of symmetry are identified. Folding an equilateral triangle highlights multiple lines of symmetry and proves that not only are all sides equal in length, but also that all interior angles have the same measure.

Module 4: *Angle Measurement and Plane Figures*

Pacing: April 29- May 31 **20 Days**		
Topic	**Lesson**	**Lesson Objective/ Supportive Videos**
Topic A: Lines and Angles	Lesson 1	Identify and draw points, lines, line segments, rays, and angles and recognize them in various contexts and familiar figures. https://www.youtube.com/watch?v=FdjnwlJTfXE&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU&index=1
	Lesson 2	Use right angles to determine whether angles are equal to, greater than, or less than right angles. Draw right, obtuse, and acute angles. https://www.youtube.com/watch?v=eISQqaAnWqg&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU&index=2
	Lesson 3	Identify, define, and draw perpendicular lines. https://www.youtube.com/watch?v=o_QojrIWyKs&index=3&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU
	Lesson 4	Identify, define, and draw parallel lines. https://www.youtube.com/watch?v=JEGIpXuXQdA&index=4&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU
Topic B: Angle Measurement	Lesson 5	Use a circular protractor to understand a 1-degree angle as 1/360 of a turn. Explore benchmark angles using the protractor. https://www.youtube.com/watch?v=7_TZsyKE4pQ&index=5&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU
	Lesson 6	Use varied protractors to distinguish angle measure from length measurement. https://www.youtube.com/watch?v=PP84Ot_wBwQ&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU&index=6
	Lesson 7	Measure and draw angles. Sketch given angle measures and verify with a protractor. https://www.youtube.com/watch?v=icvcbAWG5qM&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU&index=7
	Lesson 8	Identify and measure angles as turns and recognize them in various contexts. https://www.youtube.com/watch?v=WKCIMOxuLRM&index=8&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU
Module Assessment May 9-10, 2019		

Topic C: Problem Solving with the Addition of Angle Measures	Lesson 9	Decompose angles using pattern blocks. https://www.youtube.com/watch?v=YHg1Ofrzaj8&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU&index=9
	Lesson 10	Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure. https://www.youtube.com/watch?v=MA5A-Zk5CX4&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU&index=10
	Lesson 11	Use the addition of adjacent angle measures to solve problems using a symbol for the unknown angle measure. https://www.youtube.com/watch?v=Jj4BTjB8fWY&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU&index=11
Topic D: Two-Dimensional Figures and Symmetry	Lesson 12	Recognize lines of symmetry for given two-dimensional figures; identify line-symmetric figures and draw lines of symmetry. https://www.youtube.com/watch?v=qvfsHtVPbGE&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU&index=12
	Lesson 13	Analyze and classify triangles based on side length, angle measure or both. https://www.youtube.com/watch?v=QkF-5DSyYnE&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU&index=13
	Lesson 14	Define and construct triangles from given criteria. Explore symmetry in triangles. https://www.youtube.com/watch?v=RQP_2cuXY8M&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU&index=14
	Lesson 15	Classify quadrilaterals based on parallel and perpendicular lines and the presence or absence of angles of a specified size. https://www.youtube.com/watch?v=WkW2qeH0eY8&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU&index=15
	Lesson 16	Reason about attributes to construct quadrilaterals on square or triangular grid paper. https://www.youtube.com/watch?v=ebGJpGJLfBs&index=16&list=PLvolZqLMhJmn8fF4yoPjFSzHVVwR0JncU
End of Module Assessment May 30-31, 2019		

NJSLS Standards:

4.MD.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
4.MD.5a	An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
4.MD.5b	An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

- This standard brings up a connection between angles and circular measurement (360 degrees). Angle measure is a “turning point” in the study of geometry. Students often find angles and angle measure to be difficult concepts to learn, but that learning allows them to engage in interesting and important mathematics. An angle is the union of two rays, **a** and **b**, with the same initial point P. The rays can be made to coincide by rotating one to the other about P; this rotation determines the size of the angle between **a** and **b**. The rays are sometimes called the sides of the angles.

- Another way of saying this is that each ray determines a direction and the angle size measures the change from one direction to the other. Angles are measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and degrees are the unit used to measure angles in elementary school. A full rotation is thus 360°

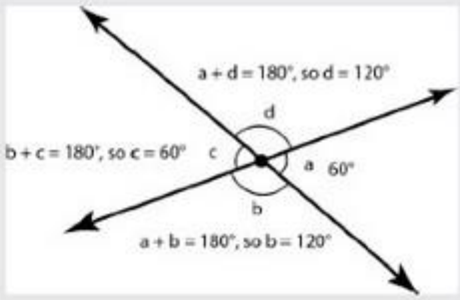
- Two angles are called complementary if their measurements have the sum of 90°. Two angles are called supplementary if their measurements have the sum of 180°. Two angles with the same vertex that overlap only at a boundary (i.e., share a side) are called adjacent angles. These terms may come up in classroom discussion, they will not be tested. This concept is developed thoroughly in middle school (7th grade).

- Like length, area, and volume, angle measure is additive: The sum of the measurements of adjacent angles is the measurement of the angle formed by their union. This leads to other important properties. If a right angle is decomposed into two adjacent angles, the sum is 90°, thus they are complementary. Two adjacent angles that compose a “straight angle” of 180° must be supplementary.

An angle

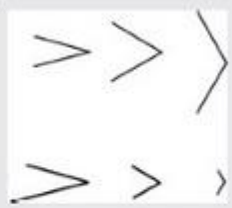
name	measurement
right angle	90°
straight angle	180°
acute angle	between 0 and 90°
obtuse angle	between 90° and 180°
reflex angle	between 180° and 360°

Angles created by the intersection of two lines



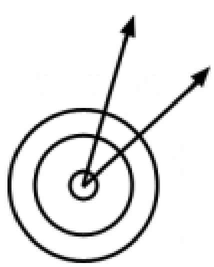
When two lines intersect, they form four angles. If the measurement of one is known (e.g., angle *a* is 60°), the measurement of the other three can be determined.

Two representations of three angles



Initially, some students may correctly compare angle sizes only if all the line segments are the same length (as shown in the top row). If the lengths of the line segments are different (as shown in the bottom row), these students base their judgments on the lengths of the segments, the distances between their endpoints, or even the area of the triangles determined by the drawn arms. They believe that the angles in the bottom row decrease in size from left to right, although they have, respectively, the same angle measurements as those in the top row.

- The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles yet the angle measure is the same.

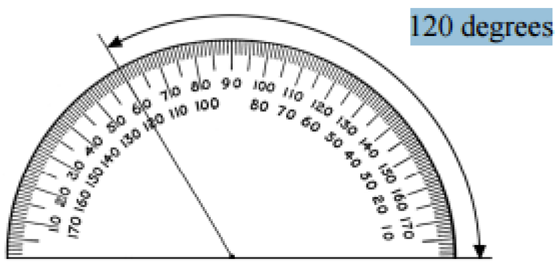


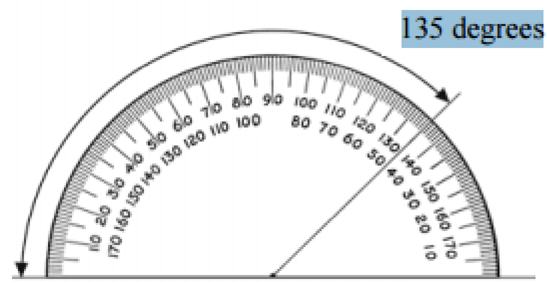
This standard calls for students to explore an angle as a series of "one-degree turns." A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100°, how many one-degree turns has the sprinkler made?

4.MD.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

- Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360° rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90° and 180°. They extend this understanding and recognize and sketch angles that measure approximately 45° and 30°.

- They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular). Students should measure angles and sketch angles.



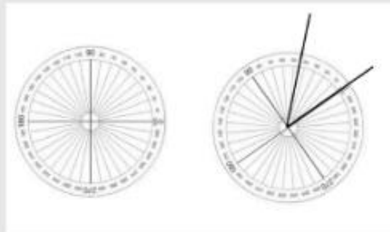


- As with all measureable attributes, students must first recognize the attribute of angle measure, and distinguish it from other attributes!

- As with other concepts students need varied examples and explicit discussions to avoid learning limited ideas about measuring angles (e.g., misconceptions that a right angle is an angle that points to the right, or two right angles represented with different orientations are not equal in measure).

- If examples and tasks are not varied, students can develop incomplete and inaccurate notions. For example, some come to associate all slanted lines with 45° measures and horizontal and vertical lines with measures of 90°. Others believe angles can be “read off” a protractor in “standard” position, that is, a base is horizontal, even if neither ray of the angle is horizontal. Measuring and then sketching many angles with no horizontal or vertical ray perhaps initially using circular 360° protractors can help students avoid such limited conceptions.

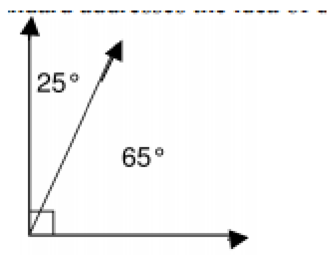
A 360° protractor and its use



The figure on the right shows a protractor being used to measure a 45° angle. The protractor is placed so that one side of the angle lies on the line corresponding to 0° on the protractor and the other side of the angle is located by a clockwise rotation from that line.

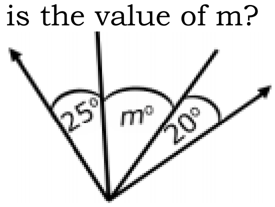
4.MD.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

- This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.



Example:
A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved? If the water sprinkler rotates a total of 25 degrees then pauses. How many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?

Example:
If the two rays are perpendicular, what is the value of m?

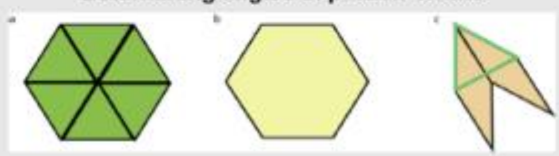


Example:
Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30°. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?

- Students can develop more accurate and useful angle and angle measure concepts if presented with angles in a variety of situations. They learn to find the common features of superficially different situations such as turns in navigation, slopes, bends, corners, and openings.

- With guidance, they learn to represent an angle in any of these contexts as two rays, even when both rays are not explicitly represented in the context; for example, the horizontal or vertical in situations that involve slope (e.g., roads or ramps), or the angle determined by looking up from the horizon to a tree or mountain-top. Eventually they abstract the common attributes of the situations as angles (which are represented with rays and a vertex,) and angle measurements.

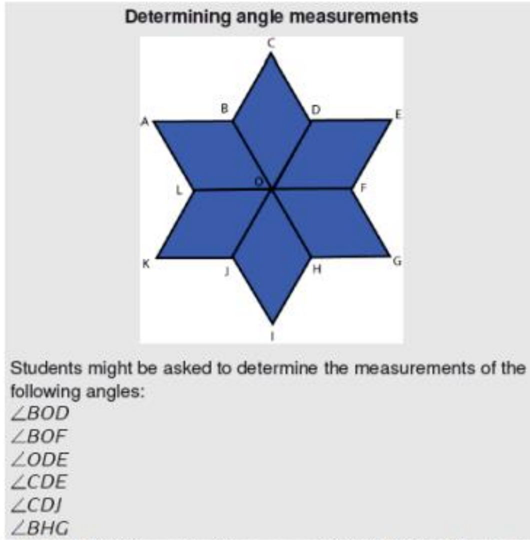
Determining angles in pattern blocks



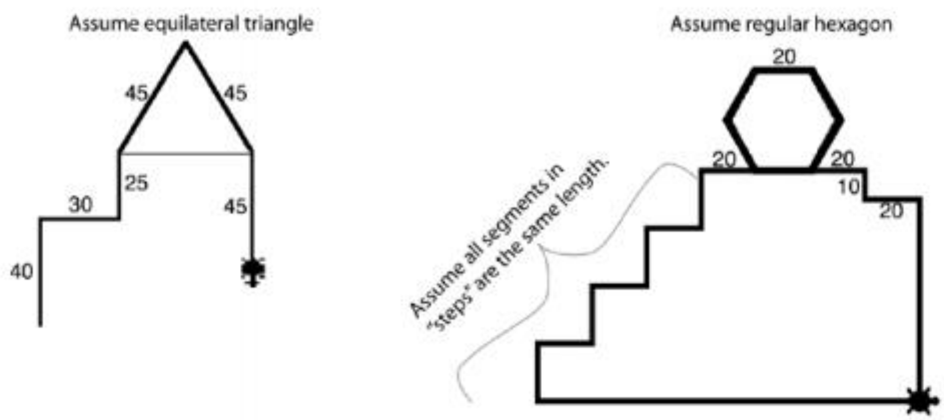
Students might determine all the angles in the common "pattern block" shape set based on equilateral triangles. Placing six equilateral triangles so that they share a common vertex (as shown in part a), students can figure out that because the sum of the angles at this vertex is 360°, each angle which shares this vertex must have measure 60°. Because they are congruent, all the angles of the equilateral triangles must have measure 60° (again, to ensure they develop a firm foundation, students can verify these for themselves with a protractor). Because each angle of the regular hexagon (part b) is composed of two angles from equilateral triangles, the hexagon's angles each measure 120°. Similarly, in a pattern block set, two of the smaller angles from tan rhombi compose an equilateral triangle's angle, so each of the smaller rhombus angles has measure 30°.

- Students with an accurate conception of angle can recognize that angle measure is additive. As with length, area, and volume, when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Students can then solve interesting and challenging addition and subtraction problems to find the measurements of unknown angles on a diagram in real world and mathematical problems.

- For example, they can find the measurements of angles formed by a pair of intersecting lines, as illustrated above, or given a diagram showing the measurement of one angle, find the measurement of its complement. They can use a protractor to check measurement, not to check their reasoning, but to ensure that they develop full understanding of the mathematics and mental images for important benchmark angles (e.g., 30°, 45°, 60°, and 90°).



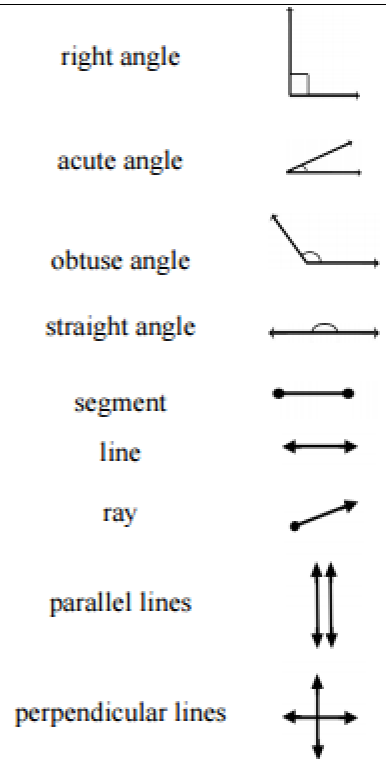
(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 24)



(*Progressions for the CCSSM, Geometric Measurement*, CCSS Writing Team, June 2012, page 25)

4.G.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

- This standard asks students to draw two-dimensional geometric objects and to also identify them in two dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines. Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. Students may not easily identify lines and rays because they are more abstract.



- Developing a clear understanding that a point, line, and plane are the core attributes of space objects, and real world situations can be used to think about these attributes. Enforcing precise geometrical vocabulary is important for mathematical communication.

Example:
How many acute, obtuse and right angles are in this shape?

- Line segments and rays are sets of points that describe parts of lines, shapes, and solids. Angles are formed by two intersecting lines or by rays with a common endpoint. They are classified by size.

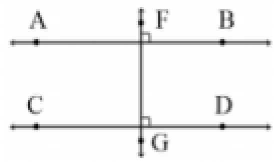
4.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

- Classify triangles based on the presence or absence of perpendicular lines and based on the presence or absence of angles of a particular size.

- Classify quadrilaterals based on the presence or absence of parallel or perpendicular lines and based on the presence or absence of angles of a particular size.

- Two-dimensional or plane shapes have many properties that make them different from one another. Students should become familiar with the concept of parallel and perpendicular lines.

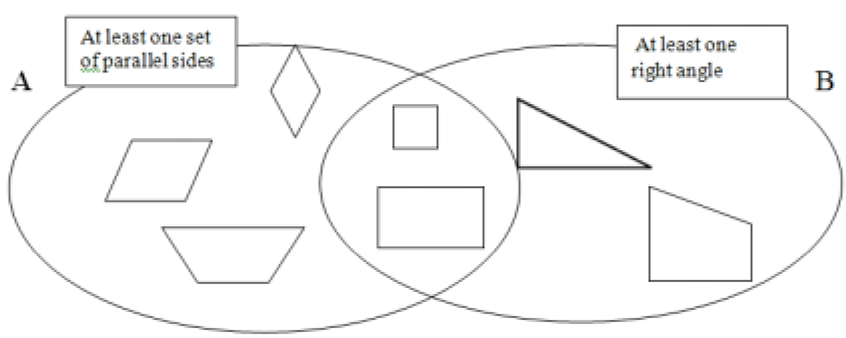
- Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90º). Parallel and perpendicular lines are shown below:



- Polygons can be described and classified by their sides and angles. Identify triangles, quadrilaterals, pentagons, hexagons, and octagons based on their attributes. Have a clear understanding of how to define and identify a right triangle.

- Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles. A kite is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside (adjacent to) each other.

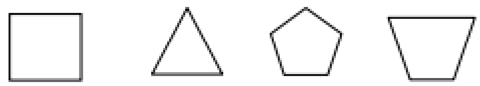
- This standard calls for students to sort objects based on parallelism, perpendicularity and angle types. Example: Which figure in the Venn diagram below is in the wrong place, explain how do you know?



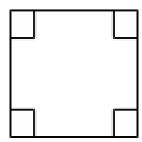
- Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles. Example: Draw and name a figure that has two parallel sides and exactly 2 right angles.

Example:
For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show a counter example.
• A parallelogram with exactly one right angle.
• An isosceles right triangle.
• A rectangle that is not a parallelogram. (impossible)
• Every square is a quadrilateral.
• Every trapezoid is a parallelogram.

Example: Identify which of these shapes have perpendicular or parallel sides and justify your selection.



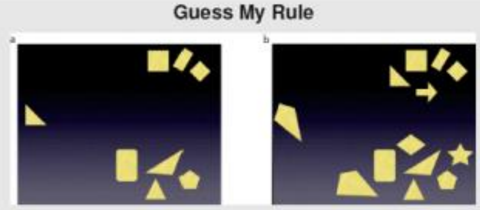
A possible justification that students might give is: The square has perpendicular lines because the sides meet at a corner, forming right angles



- Angle Measurement: This expectation is closely connected to 4.MD.5, 4.MD.6, and 4.G.1. Students' experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles.

- Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides.

Guess My Rule



Students can be shown the two groups of shapes in part a and asked "Where does the shape on the left belong?" They might surmise that it belongs with the other triangles at the bottom. When the teacher moves it to the top, students must search for a different rule that fits all the cases.

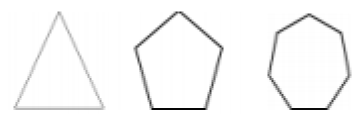
Later (part b), students may induce the rule: "Shapes with at least one right angle are at the top." Students with rich visual images of right angles and good visualization skills would conclude that the shape at the left (even though it looks vaguely like another one already at the bottom) has one right angle, thus belongs at the top.

- The notion of congruence ("same size and same shape") may be part of classroom conversation but the concepts of congruence and similarity do not appear until middle school

- **TEACHER NOTE:** In the U.S., the term "trapezoid" may have two different meanings. Re-

search identifies these as inclusive and exclusive definitions. The inclusive definition states: A trapezoid is a quadrilateral with at least one pair of parallel sides. The exclusive definition states: A trapezoid is a quadrilateral with exactly one pair of parallel sides. With this definition, a parallelogram is not a trapezoid. (Progressions for the CCSSM: Geometry, June 2012.)	
4.G.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

- Gain a conceptual understanding that a line of symmetry will split a figure into two equal parts.
- Recognize a line of symmetry for a two-dimensional figure as a line across the figure, so that the figure can be folded along the line into matching parts.
- Develop an understanding that each half of a figure is a mirror image of the other half. Draw lines of symmetry. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry.
- Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex.



Common multiplication and division situations. [1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	**3 x ? = 18**, and **18 ÷ 3 = ?**	**? x 6 = 18**, and **18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS [2], AREA [3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	**a x b = ?**	**a x ? = p** and **p ÷ a = ?**	**? x b = p**, and **p ÷ b = ?**

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Module 4 Assessment / Authentic Assessment Recommended Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Authentic Assessment	4.MD.5	30 mins	Individual
Optional Mid-Module Assessment	4.MD.5 4.MD.6 4.G.1	1 Block	Individual
Optional End of Module Assessment	4.MD.5 4.MD.6 4.MD.7 4.G.1 4.G.2 4.G.3	1 Block	Individual

Fourth Grade Ideal Math Block

Getting Ready: Whole Group — 7-10 min.
Number Talk/ Number Strings

Fluency: Whole Group
Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

Concept Development: Individual/partner/whole
Instruction & Strategic Problem Set Questions

Student Debrief: Independent
Exit Ticket

(Fluency through Student Debrief: 50-60 min.)

CENTERS/STATIONS:
Pairs / Small Group/ Individual — 15-20 min.
DIFFERENTIATED activities designed to **RETEACH, REMEDIATE, ENRICH** student's understanding of concepts.

M:	A:	T:	H:
Meet with the teacher	**Application/ Problem Solving**	**Technology**	**Hands on Activities**

SUMMARY: Whole Group — Up to 5 min.
Lesson Closure: Student Reflection; Real Life Connections to Concept

Eureka Lesson Structure:

Fluency:

- Sprints
- Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Concept Development: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Student Debrief:

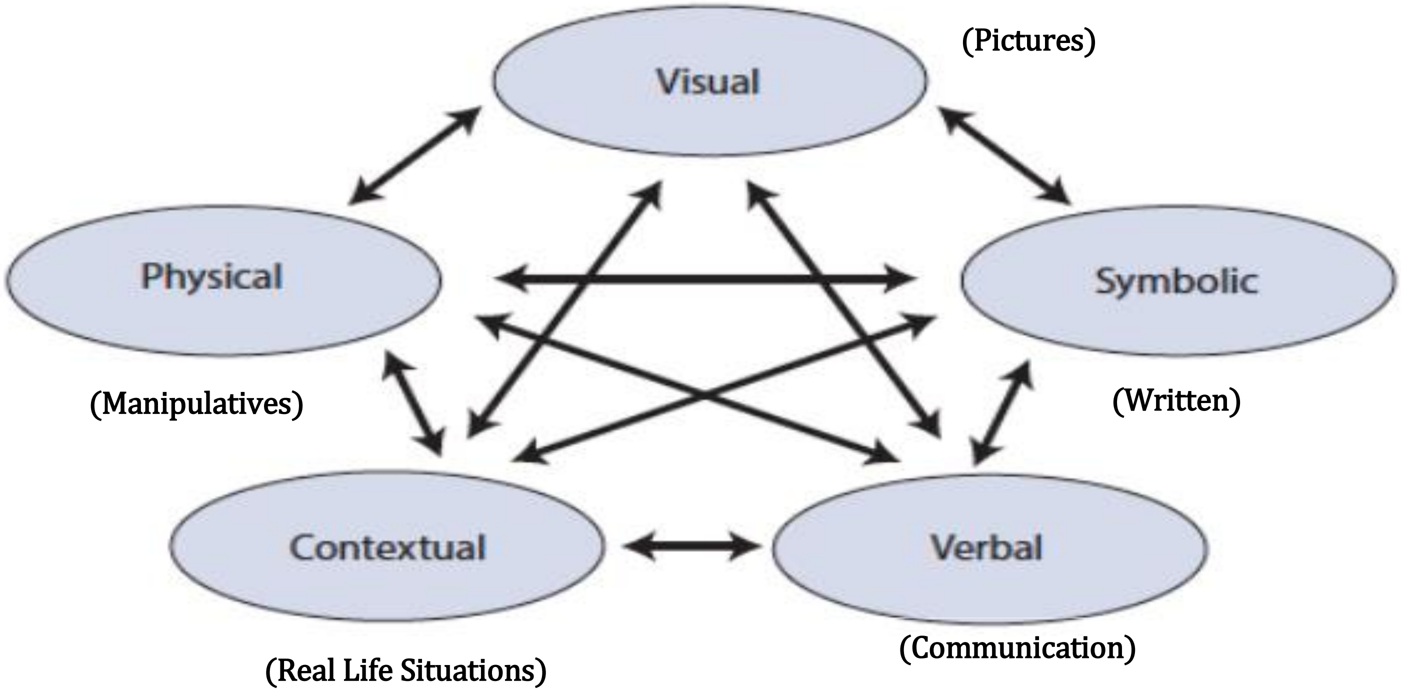
- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

PARCC Assessment Evidence/Clarification Statements			
CCSS	**Evidence Statement**	**Clarification**	**MP**
4.MD.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.		MP 2
4.MD.6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. -		MP 2, 5
4.MD.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. -		MP 1,7
4.G.1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two dimensional figures.		MP 5
4.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	• A trapezoid is defined as “A quadrilateral with at least one pair of parallel sides.” • Tasks may include terminology: equilateral, isosceles, scalene, acute, right, and obtuse.	MP 7
4.G.3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. - -		

Student Name: ______________________________ Task: _____________________ School: ________ Teacher: __________ Date: ________

“I CAN.....”	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that’s it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a class-mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my think-ing.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.
Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.
Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.
DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?
WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

100 questions that promote Mathematical Discourse

Help students **work together** to make sense of mathematics

1. What **strategy** did you use?
2. Do you **agree**?
3. Do you **disagree**?
4. Would you **ask the rest of the class** that question?
5. Could you **share your method** with the class?
6. What part of what he said **do you understand**?
7. Would someone like to **share** ___?
8. Can you **convince the rest of us** that your answer makes sense?
9. **What do others think** about what [student] said?
10. Can someone **retell or restate** [student]'s explanation?
11. Did you **work together**? In what way?
12. Would anyone like to **add to what was said**?
13. Have you **discussed** this with your group? With others?
14. Did anyone get a **different answer**?
15. **Where** would you go for **help**?
16. **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
17. How could you help another student **without telling them the answer**?
18. **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

19. Is this a **reasonable answer**?
20. Does that make **sense**?
21. **Why** do you think that? Why is that true?
22. Can you **draw a picture or make a model** to show that?
23. **How** did you reach that conclusion?
24. Does anyone want to **revise** his or her answer?
25. **How were you sure** your answer was right?

Ready

Help students learn to **reason mathematically**

26 **How did you begin** to think about this problem?

27 What is **another way** you could solve this problem?

28 How could you **prove** ______?

29 Can you **explain how your answer is different from or the same as** [student]'s answer?

30 Let's **break the problem into parts**. What would the parts be?

31 Can you **explain this part more specifically**?

32 Does that **always work**?

33 Can you think of a case where that **wouldn't work**?

34 How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35 What do you need to do **next**?

36 What have you **accomplished**?

37 What are your **strengths and weaknesses**?

38 Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39 What is this problem about? What can you **tell me about it**?

40 Do you need to **define or set limits** for the problem?

41 How would you **interpret** that?

42 Could you **reword that in simpler terms**?

43 Is there something that can be **eliminated** or that is **missing**?

44 Could you **explain** what the problem is asking?

45 What **assumptions** do you have to make?

46 What do you **know** about this part?

47 Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74 What is the **relationship** between ___ and ___?

75 Have we ever solved a problem **like this before**?

76 What uses of mathematics did you find in the **newspaper** last night?

77 What is the **same**?

78 What is **different**?

79 Did you use skills or build on concepts that were **not necessarily mathematical**?

80 Which **skills or concepts** did you use?

81 What **ideas** have we explored before that were useful in solving this problem?

82 Is there a **pattern**?

83 **Where else** would this strategy be useful?

84 How does this **relate** to ___?

85 Is there a **general rule**?

86 Is there a **real-life situation** where this could be used?

87 How would your method work with **other problems**?

88 What other problem does this seem to **lead to**?

Help students **persevere**

89 Have you tried making a **guess**?

90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

93 Give me another **related problem**. Is there an easier problem?

94 How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95 What was **one thing you learned** (or two, or more)?

96 Did you **notice any patterns**? If so, describe them.

97 What **mathematics topics** were used in this investigation?

98 What were the **mathematical ideas** in this problem?

99 What is mathematically **different about these two situations**?

100 What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation

3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)
3.NBT.A.2: Add/subtract within 1000
4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm
5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

Mathematical Proficiency

To be mathematically proficient, a student must have:

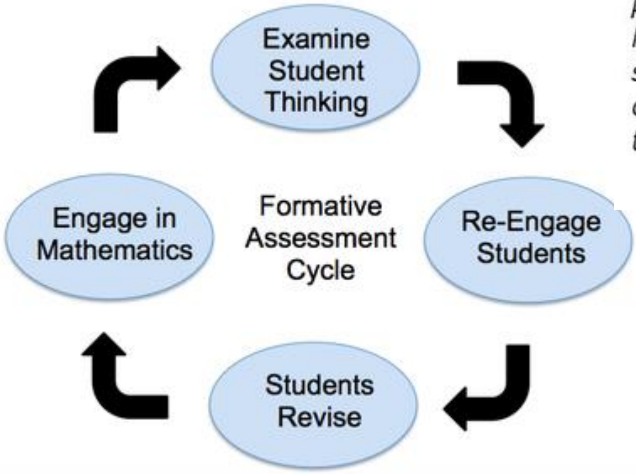
- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)



Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The Standards for Mathematical Practice:

Describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1	**Make sense of problems and persevere in solving them** Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2	**Reason abstractly and quantitatively** Mathematically proficient fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3	**Construct viable arguments and critique the reasoning of others** In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4	**Model with mathematics** Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evalu-

	ate their results in the context of the situation and reflect on whether the results make sense.
5	**Use appropriate tools strategically** Mathematically proficient fourth graders consider the available tools(including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
6	**Attend to precision** As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7	**Look for and make use of structure** In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
8	**Look for and express regularity in repeated reasoning** Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.

In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.
In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: ____ Time: __________

NJSLS.:

____

____

Objective(s): By the end of this task, I will be able to:

- ____
- ____
- ____

Task(s):

- ____
- ____
- ____
- ____

Exit Ticket:

- ____
- ____
- ____

MATH WORKSTATION SCHEDULE **Week of: ________________**

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OBSERVATIONAL DATA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

Fourth Grade PLD Rubric

Got It Evidence shows that the student essentially has the target concept or big math idea.		**Not There Yet** Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% **Distinguished command**	**PLD Level 4: 89%** **Strong Command**	**PLD Level 3: 79%** **Moderate Command**	**PLD Level 2: 69%** **Partial Command**	**PLD Level 1: 59%** **Little Command**
Student work shows **distinguished levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes an **efficient and logical progression** of mathematical reasoning and understanding.	Student work shows **strong levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes a **logical progression** of mathematical reasoning and understanding.	Student work shows **moderate levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes a **logical but incomplete progression** of mathematical reasoning and understanding. Contains **minor errors.**	Student work shows **partial understanding** of the mathematics. Student **constructs** and **communicates** an **incomplete response** based on student's attempts of explanations/ reasoning using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes an **incomplete or illogical progression** of mathematical reasoning and understanding.	Student work shows **little understanding** of the mathematics. Student **attempts** to **constructs** and **communicates** a response using the: • Tools: ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary Response includes **limited evidence of the progression** of mathematical reasoning and understanding.
5 points	**4 points**	**3 points**	**2 points**	**1 point**

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

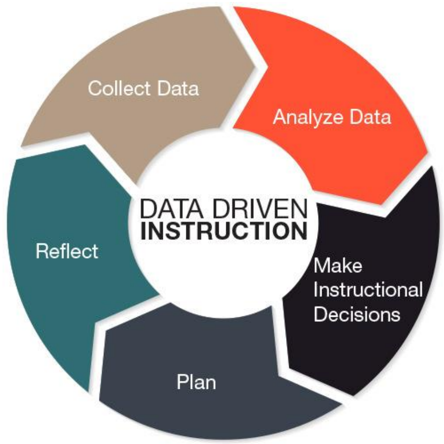
Answering these questions will help inform instructional decisions and will influence lesson planning.

Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?



Now it is time to begin the analysis again.

Data Analysis Form **School:** ________________ **Teacher:** ________________________ **Date:** ______________

Assessment: ____ **NJSLS:** ____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

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MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews[1].
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended.*
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)[2].
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

GRADES K-2

Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students <u>should retain ALL of their current artifacts</u> in their Mathematics Portfolio

4TH Grade Authentic Performance Task: Matthew and Nick's Circles

Matthew and Nick were investigating angles and circles, drawing circles and creating angles inside of their circles.

Matthew drew a small circle and divided it into six equal sections. He measured the angles of each section and found that they were all 60º.

Nick decided to draw a circle that was larger than Matthew's circle. He divided his circle into six equal sections and measured the angles of each section. He expected them to be larger than 60º, but they all measured 60º.

The resource sheet **Circles and Angles** shows the work that Matthew and Nick did.

Why might Nick have thought the sections of his circle would have a larger angle measurement than the sections in Matthew's circle?

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Why do the sections in Nick's circle and the sections in Matthew's circle have the same angle measurement?

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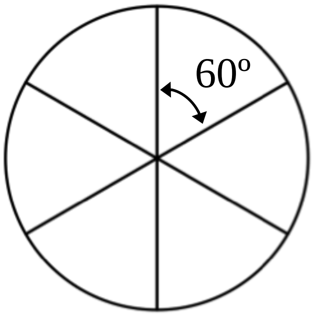
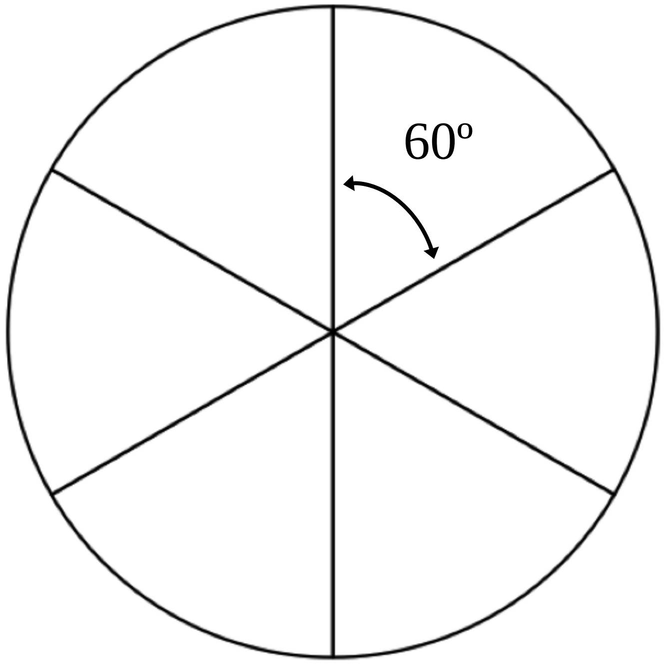
Circles and Angles
Resource Sheet

Matthew's circle

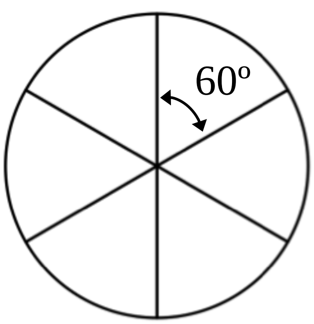


Nick's circle

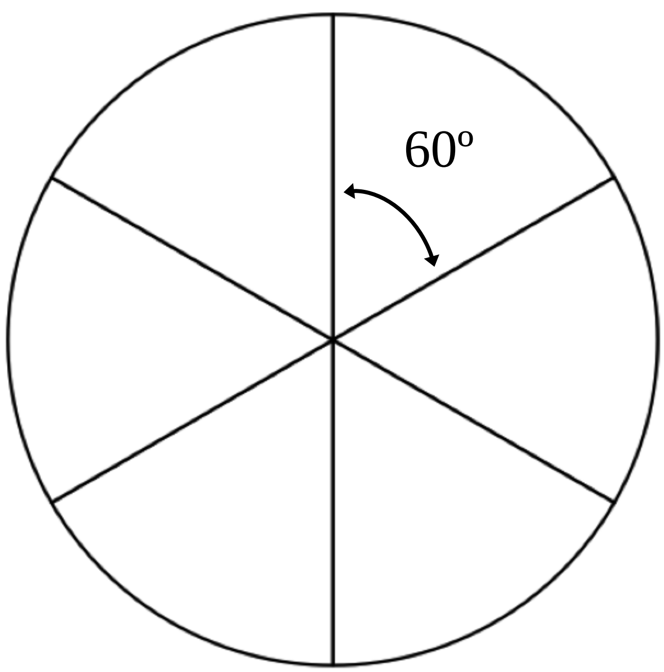


Circles and Angles
Resource Sheet

Matthew's circle



Nick's circle



4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

SOLUTION:

See below

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on explanations/reasoning using the: • parts of an angle and define what an angle is • A circle is 360 degrees • Understand that an angle that turns through 1/360 of a circle is a 1 degree angle Response includes an **efficient** and logical progression of steps.	Clearly constructs and communicates a complete response based on explanations/ reasoning using the: • parts of an angle and define what an angle is. • A circle is 360 degrees • Understand that an angle that turns through 1/360 of a circle is a 1 degree angle Response includes a **logical** progression of steps	Constructs and communicates a complete response based on explanations/ reasoning using the: • parts of an angle and define what an angle is. • A circle is 360 degrees • Understand that an angle that turns through 1/360 of a circle is a 1 degree angle Response includes a **logical but incomplete** progression of steps. Minor calculation errors.	Constructs and communicates an incomplete response based on explanations/ reasoning using the: • parts of an angle and define what an angle is. • A circle is 360 degrees • Understand that an angle that turns through 1/360 of a circle is a 1 degree angle Response includes an **incomplete or Illogical** progression of steps.	The student shows no work or justification

Resources

Engage NY
http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

Common Core Tools
http://commoncoretools.me/
http://www.ccsstoolbox.com/
http://www.achievethecore.org/steal-these-tools

Achieve the Core
http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000

http://www.thinkingblocks.com/

Illustrative Math Project :http://illustrativemathematics.org/standards/k8

Inside Mathematics: http://www.insidemathematics.org/index.php/tools-for-teachers

Sample Balance Math Tasks: http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/

Georgia Department of Education:https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx

Gates Foundations Tasks:http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf

Minnesota STEM Teachers' Center:
http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships

Singapore Math Tests K-12: http://www.misskoh.com

Mobymax.com: http://www.mobymax.com

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .